1. Find a way to calculate

$$f(x) = x - \sqrt{x^2 - a}$$

correctly to the number of digits used when $x$ is very much larger than $a$.

[1 mark]

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Solution:

$$\left(x - \sqrt{x^2 - a}\right) \cdot \frac{\left(x + \sqrt{x^2 - a}\right)}{\left(x + \sqrt{x^2 - a}\right)}$$

$$= \frac{x^2 - \left(x^2 - a\right)}{x + \sqrt{x^2 - a}}$$

$$= \frac{a}{x + \sqrt{x^2 - a}}$$

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2. For a function $f(x)$, the Newton's divided-difference formula gives the interpolating polynomial for a set of 4 data points as

$$P_3(x) = 1 + 4x + 4x(x-0.25) + \frac{16}{3}x(x-0.25)(x-0.5).$$

(a) Find $f(0.5)$ and the respective relative error.

(b) If one of the data points used is 0.75, construct the divided difference table.

[5 marks]

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Solution:

(a) Based on the Newton's divided-difference formula, the node points used to construct the interpolating polynomial is 0, 0.25, 0.5 and $x$.

Since 0.5 is one of the node points used, thus

$$f(0.5) = P_3(0.5) = 3.5$$

with the relative error is 0.

(b) Given that $x$ = 0.75, the divided difference table is then constructed.

| $x_i$ | $f[x_i]$ | $f[x_i, x_{i+1}]$ | $f[x_i, x_{i+1}, x_{i+2}]$ | $f[x_i, x_{i+1}, x_{i+2}, x_{i+3}]$ |
|---|---|---|---|---|
| 0 | 1 | | | |
| | | 4 | | |
| 0.25 | 2 | | 4 | |
| | | 6 | | 16/3 |
| 0.50 | 3.5 | | 8 | |
| | | 10 | | |
| 0.75 | 6 | | | |