<u>Solution to Assignment 2</u>

For σ_1, we have $\underset{\sim}{\mathbf{n}} = -\underset{\sim}{\mathbf{k}}$, so $\underset{\sim}{\mathbf{F}} \cdot \underset{\sim}{\mathbf{n}} = \underset{\sim}{\mathbf{F}} \cdot \left(-\underset{\sim}{\mathbf{k}}\right) = -x^2 z - y^2 = -y^2$ (since $z = 0$ on σ_1). So, if D is the unit disk, we get

$$\iint_{\sigma_1} \underset{\sim}{\mathbf{F}} \cdot d\underset{\sim}{\mathbf{S}} = \iint_{\sigma_1} \underset{\sim}{\mathbf{F}} \cdot \underset{\sim}{\mathbf{n}}\, dS = \iint_{D_1} \left(-y^2\right) dA$$

$$= -\int_0^{2\pi} \int_0^1 r^2 \sin^2\theta \, r \, dr \, d\theta$$

Use cylindrical coordinates

$$= -\frac{\pi}{4}$$

Now, since σ_2 is closed, we can use Divergence Theorem. Since $div\,\underset{\sim}{\mathbf{F}} = z^2 + y^2 + x^2$, if G is the solid bounded by the unit upper hemisphere and the unit disk on *xy*-plane, we use spherical coordinates to get

$$\iint_{\sigma_2} \underset{\sim}{\mathbf{F}} \cdot d\underset{\sim}{\mathbf{S}} = \iiint_G div\,\underset{\sim}{\mathbf{F}}\, dV$$

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^1 \rho^2 \rho^2 \sin\phi \, d\rho \, d\phi \, d\theta$$

$$= \frac{2\pi}{5}$$

Finally,

$$\iint_{\sigma} \underset{\sim}{\mathbf{F}} \cdot d\underset{\sim}{\mathbf{S}} = \iint_{\sigma_2} \underset{\sim}{\mathbf{F}} \cdot d\underset{\sim}{\mathbf{S}} - \iint_{\sigma_1} \underset{\sim}{\mathbf{F}} \cdot d\underset{\sim}{\mathbf{S}}$$

$$= \frac{2\pi}{5} - \left(-\frac{\pi}{4}\right)$$

$$= \frac{13\pi}{20}$$