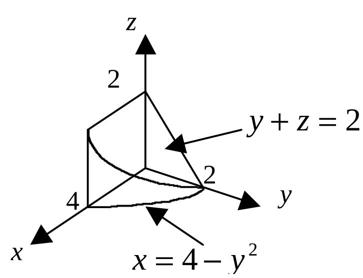
Multimedia University
Faculty of Engineering
PEM2036 Engineering Mathematics III
Mid-Trimester Test Solutions 15 June 2001 1.5 hours

ID:

1. Change the Cartesian integral $\int_{-a}^{a}\int_{-\sqrt{a^2-x^2}}^{\sqrt{a^2-x^2}} dy\,dx$ into an equivalent polar integral.

Answer: $\underline{\int_0^{2p}\int_0^{a} r\ dr\,dq}$

2. Which of the following integral does not represent the volume of the region in the first octant bounded by the coordinate planes, the plane $y+z=2$, and the cylinder $x=4-y^2$.



Answers:

$\underline{\int_0^4\int_0^{\sqrt{4-z}}\int_0^{2-y} dz\,dy\,dx}$ **OR/AND** $\underline{\int_0^2\int_{2-z}^{2}\int_0^{4-y^2} dx\,dy\,dz}$

3. A circular cylinder hole is bore through the center of a solid sphere. The volume of the remaining solid is $V=2\int_0^{2p}\int_0^{\sqrt{3}}\int_1^{\sqrt{4-z^2}} r\ dr\,dz\,dq$. Find the radius of the hole and the radius of the sphere.

Answer: <u>radius of hole = 1, radius of sphere = 2</u>

4. Find the value of the Jacobian $\dfrac{\partial(x,y,z)}{\partial(u,v,w)}$ for the system $x=2u-w$, $y=u+3v$ and $z=v+3w$.

Answer: <u>17</u>

5. Given that $\int_C \underset{\sim}{\mathbf{F}}\cdot d\underset{\sim}{\mathbf{r}}$ s independent of path for every piecewise smooth curve *C*. Which of the following is equivalent to the above statement?

(i) $\int_C \underset{\sim}{\mathbf{F}}\cdot d\underset{\sim}{\mathbf{r}}=0$ for every piecewise smooth closed curve *C*.

(ii) $\underset{\sim}{\mathbf{F}}$ is a conservative vector field.

(iii) There exists a function f such that $\underset{\sim}{\mathbf{F}}=\nabla(f+k)$ where *k* is a constant.

Answer: <u>i, ii, iii</u>

6. Which of the following field is conservative? **Answer**: $\underline{2xy\,\underset{\sim}{\mathbf{i}} + \left(x^2 - 3y^2\right)\underset{\sim}{\mathbf{j}}}$

7. Apply Green's Theorem to evaluate the integral

$$\oint_C (6y + x)dx + (y + 2x)dy$$

where C: the circle $x^2 + y^2 = 4$. **Answer**: $\underline{-16p}$

8. Find the volume under the plane $z = 2x + 1$ and over the rectangle

$$R = \{(x, y) : 3 \le x \le 5, 1 \le y \le 2\}.$$

Answer: $\underline{18}$

9. Consider the double integrals $\int_{-1}^{3}\int_{2}^{5} -\sin u \, dx \, dy$. Then which of the followings is true?

Answer: The domain of integration is $R = \{(x, y) : -1 \le y \le 3, 2 \le x \le 5\}$.
(*Note: u is a constant here.*)

10. The domain R of $\int_0^1 \int_x^1 \frac{1}{1+x^2} \, dy \, dx$ is

Answer: R is a triangular domain with vertices (0, 0), (1, 1) and (0, 1).

11. The double integrals that gives the volume of the solid under the paraboloid $z = x^2 + y^2$ and above the region bounded by $y = x^2$ and $x = y^2$ is

Answer: $\underline{\int_0^1 \int_{y^2}^{\sqrt{y}} x^2 + y^2 \, dx \, dy}$

12. The transformation resulting from $x = r\cos q$, $y = r\sin q$ will map the set $\{(x, y) : 4 \le x^2 + y^2 \le 16\}$ to

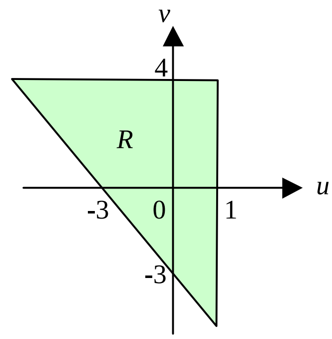
Answer: $\underline{\{(r, q) : 2 \le r \le 4, 0 \le q \le 2p\}}$

13. Evaluate $\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} \left(x^2 + y^2\right)^{\frac{3}{2}} dy \, dx$. **Answer:** $\underline{\frac{p}{5}}$

14. Evaluate $\int_0^1 \int_0^{1-y} \int_0^2 dx \, dz \, dy$. **Answer:** $\underline{1}$

15. The transformation $u = x - 2y$, $v = 2x + y$ will map the triangular region enclosed by $x - 2y = 1$, $2x + y = 4$ and $3x - y = -3$ to the region R as shown below.

Answer:



16. Evaluate the line integral $\int_C y^2\,dx$ where C is the straight line path from (0, 0) to (2, 2).

Answer: $\dfrac{8}{3}$

17. Let C be the ellipse $4x^2 + y^2 = 4$. Then Green's Theorem is not applicable to the integral $\int_C \dfrac{y}{x^2+y^2}\,dx - \dfrac{x}{x^2+y^2}\,dy$ because

Answer: The functions $\dfrac{y}{x^2+y^2}$ and $\dfrac{x}{x^2+y^2}$ are not continuous at the origin.

18. If $f(x, y, z) = 2xz^4 - x^2 y$, find ∇f at the point (2, –2, –1). **Answer:** $10\,\underset{\sim}{\mathbf{i}} - 4\,\underset{\sim}{\mathbf{j}} - 16\,\underset{\sim}{\mathbf{k}}$

19. The equation for the tangent plane to the surface $x^2 + y^2 + z^2 = 3$ at the point (1, 1, 1) is

Answer: $x + y + z = 3$

20. The curl of the vector field $(2x - y^2)\underset{\sim}{\mathbf{i}} + (3z + x^2)\underset{\sim}{\mathbf{j}} + (4y - z^2)\underset{\sim}{\mathbf{k}}$ is

Answer: $\underset{\sim}{\mathbf{i}} + 2(x + y)\underset{\sim}{\mathbf{k}}$

End of Paper.