Solutions for questions 1,2 and 3

1

(a) $v_3 = 5v_1/(15 + 5) = v_1/4$, $v_2 = v_1 - \mu v_3 = (4 - \mu)v_1/4$

(b) $v_2 = (4 - \mu)20/4 = 20 - 5\mu$, $i = v_2/5 + v_1/(15 + 5) = 5 - \mu$

$v = 4i + v_1 = 40 - 4\mu$, $R_{eq} = v/i = (40 - 4\mu)/(5 - \mu)$

(c) $\mu = 0 \Rightarrow R_{eq} = 40/5 = 8\ \Omega$

$v_c = 0 \Rightarrow$ short circuit so $R_{eq} = 4 + 5 \| (15 + 5) = 8\ \Omega$

(d) $i = 60/(6 + R_{eq})$, $v = R_{eq}i = 60R_{eq}/(6 + R_{eq})$

$p = vi = 3600R_{eq}/(6 + R_{eq})^2$

μ	$R_{eq}(\Omega)$	i(A)	v(V)	p(W)
1	9	4	36	144
5	∞	0	60	0
6	-16	-6	96	-576
7	-6	∞	$-\infty$	$-\infty$
9	-1	12	-12	-144
10	0	10	0	0

The model is invalid for $\mu = 7$, which requires infinite current from both sources.

2

(a) With $R_L = 0$: $v_{out} = 0$ so $i_{in} = v_{in}/200$

$i_{out} = i_{in} + 49i_{in} = 50(v_{in}/200) \Rightarrow i_{out}/v_{in} = 0.25$

With $R_L = \infty$: $i_{out} = 0$ so $v_{out} = 36(i_{in} + 49i_{in}) = 1800i_{in}$

$i_{in} = (v_{in} - v_{out})/200$, $v_{out} = 1800(v_{in} - v_{out})/200$

$v_{out} = 9v_{in} - 9v_{out} \Rightarrow v_{out}/v_{in} = 0.9$

(b) $i_{out\text{-}sc} = 0.25v_{in}$, $v_{out\text{-}oc} = 0.9v_{in}$. $R_t = v_{out\text{-}oc}/i_{out\text{-}sc} = 3.6\ \Omega$

With $v_{in} = 0$, $v_{out} = v_t$, and $i_{out} = -i_t$: $i_{in} = -v_t/200$

$i_t = v_t/36 - 49i_{in} - i_{in} = v_t/36 - 50(-v_t/200) = 10v_t/36$

$R_t = v_t/i_t = 36/10 = 3.6\ \Omega$

(c) $v_{out\text{-}oc} = 0.9v_{in} = 9$ V, $R_t = 3.6\ \Omega$,

$v_{out} = R_Lv_{out\text{-}oc}/(R_t + R_L) = 6.21$ V, $p_{out} = v_{out}^2/R_L = 4.82$ W

$i_{in} = (v_{in} - v_{out})/200 = 0.019$ A, $p_{in} = v_{in}i_{in} = 0.190$ W

$p_{out}/p_{in} = 4.82/0.19 = 25.4$

3

(a) (i) $v(t) = \cos^2 0.125\pi t$

$= \frac{1}{2}[1 + \cos 0.25\pi t]$

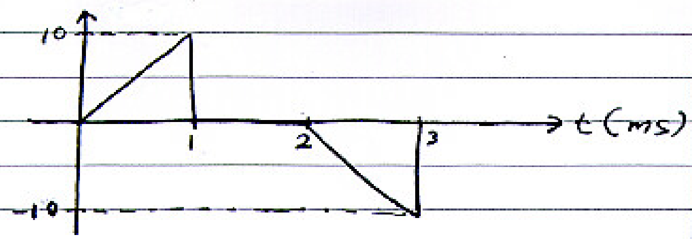
$= \frac{1}{2} + \frac{1}{2}\cos 0.25\pi t$

$V_{avg} = \frac{1}{2}$ V $\quad V_{rms} = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{1/2}{\sqrt{2}}\right)^2} = 0.6124$ V

(ii) $i(t) = 12.5\sin\left(\frac{3\pi}{4}\right)t$

$i_{avg} = 0$ A $\quad i_{rms} = \frac{12.5}{\sqrt{2}} = 8.84$ A

3 (b)



Average:

$$v(t) = \begin{cases} 10t & 0<t<1 \\ 0 & 1<t<2 \\ 20-10t & 2<t<3 \end{cases}$$

$$V_{avg} = \frac{1}{3}\left[\int_0^1 10t + \int_2^3 (20-10t)\,dt\right]$$

$$= \frac{1}{3}\left[5\left[t^2\right]_0^1 + 20\left[t\right]_2^3 - 5\left[t^2\right]_2^3\right]$$

$$= \frac{1}{3}\left[5 + 20 - 5(5)\right] = \underline{0 \ V}$$

RMS

$$v^2(t) = \begin{cases} 100t^2 & 0<t<1 \\ 0 & 1<t<2 \\ 400+100t^2-400t & 2<t<3 \end{cases}$$

$$V_{ms} = \frac{1}{3}\left[\frac{100}{3}[1] + 400(1) + \frac{100}{3}(27-8) - 200(9-4)\right]$$

$$= \frac{1}{3}\left[\frac{100}{3} + 400 + \frac{1900}{3} - 1000\right]$$

$$= 22.22$$

$$\therefore V_{rms} = \sqrt{22.22} = \underline{4.714V}$$

The Solution of Question 4

When the current source is zeroed, as shown in Figure 4 (b), Z_T and I_T are calculated as:

$$Z_T = 18\angle{-90^0}\Omega + \frac{(20\angle 90^0\Omega)(31.6\angle 71.6^0\Omega)}{51\angle 78.7^0\Omega}$$

$$= 18\angle{-90^0}\Omega + 12.4\angle 82.9^0\Omega = -j18\Omega + 1.53\Omega + j12.3\Omega = 1.53\Omega - j5.69\Omega$$

$$= 5.9\angle{-74.9^0}\Omega$$

$Z_T = 5.9\angle{-74.9^0}\Omega$

$$I_T = V_S / Z_T = I_{C(VS)} = \frac{12\angle 30^0\text{ V}}{5.9\angle{-74.9^0}\Omega} = 2.04\angle 105^0\text{ A} = (-0.522 + j1.97)\text{A}$$

$I_T == (-0.522 + j1.97)\text{A}$

When the voltage source is zeroed as shown from Figure 4 (c), the impedance (Z) of L_1, L_2, and C branch will be computed as follows:

$$Z = j30\Omega + \frac{(20\angle 90^0\Omega)(18\angle{-90^0}\Omega)}{J2\angle 90^0\Omega} = j30\Omega + \frac{360\angle 0^0\Omega}{2\angle 90^0\Omega=} = j30\Omega - j180\Omega =- j150\Omega$$

$Z =- j150\Omega$

$$I_{L2} = \frac{(10\angle 0^0\Omega)}{(10\Omega - j150\Omega)} * 500\angle 120^0\text{mA} = \frac{(10\angle 0^0\Omega)}{(150.3\angle 86.2^0)} * 500\angle 120^0\text{mA} = 33.3\angle 206^0\text{mA}$$

$$I_C(I_S) = \frac{(20\angle 90^0\Omega)}{(2\angle 90^0\Omega)} * 33.3\angle 206^0\text{mA} = 333\angle 206^0\text{mA} = (298 - j147)\text{mA}$$

The Total capacitor current is:

$I_{C(tot)} = I_C(V_S) = (0.821 - j1.82)\text{A} = 2.00\angle 114^0\text{mA}$

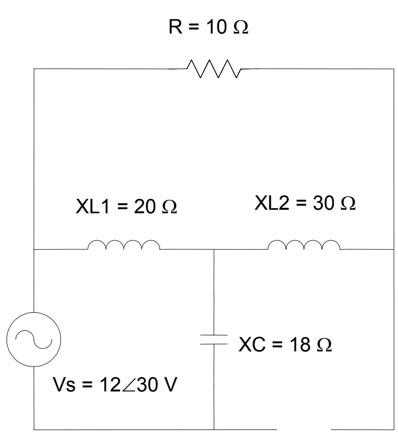


Figure 4 (b)

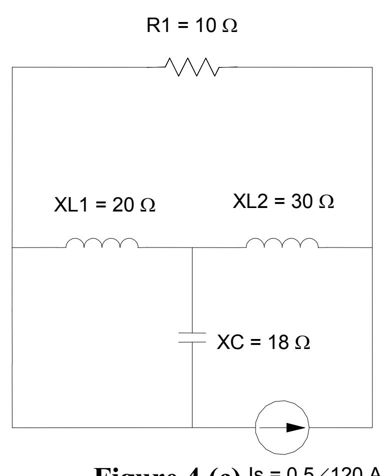


Figure 4 (c)

Solution of Question 5

The original circuit is redrawn in Figure 5 (b) for simplification combining R_1, R_2 and X_L

$$Z_A = R_1 + R_2 \| X_L = 1<0^0 K\Omega + \frac{(3.3<0^0)(3<90^0)K\Omega}{(3.3\ K\Omega + j3\ K\Omega)} = 2.4\ K\Omega + j1.64 K\Omega$$

$$= 2.98<33.4^0 K\Omega$$

$Z_A = 2.98<33.4^0 K\Omega$

Combining R_3 and Z_4, we get:

$$Z_B = R_3 \| Z_A = \frac{(10<0\ K\Omega)(2.98<33.4^0 K\Omega)}{12.62<7.5^0 K\Omega} = 2.37<25.9^0 K\Omega$$

Combining X_C and Z_B, we find:

$$Z_{TH} = X_C \| X_B = \frac{(5<-90^0 K\Omega)(2.37<25.9^0 K\Omega)}{4.52<-62.3^0 K\Omega} = (2.62<-1.8^0 K\Omega)$$

$$= 2.62\ K\Omega - j0.08\ K\Omega$$

$Z_{TH} = (2.62\ - j0.08) K\Omega$

$$V_{TH} = \frac{(Z_B)^* V_S}{(X_C + Z_{TH})} = \frac{(2.37<25.9^0 K\Omega)^* 50<0^0}{4.52<62.3^0 K\Omega} = 26.3<87.6^0 V$$

The Thevenin Circuit with R_4 connected is shown in Figure 5 (c).

$$V_{R4} = \frac{(R_4)^* V_{TH}}{(R_4 + Z_{TH})} = \frac{(4.7<0^0 K\Omega)^* 26.3<87.6^0 V}{7.32<-0.64^0 K\Omega)} = 16.9<88.2^0 V$$

$V_{R4} = 16.9<88.2^0 V$

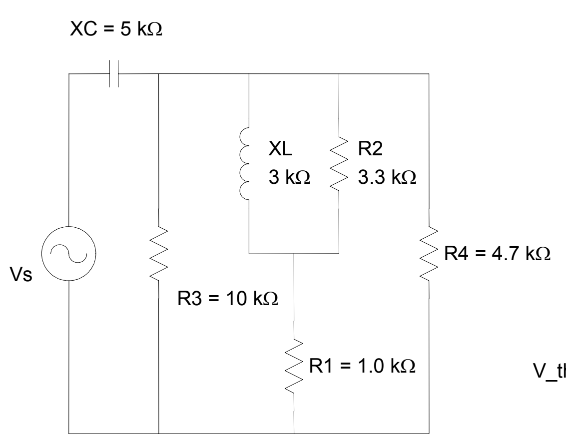


Figure 5 (b)

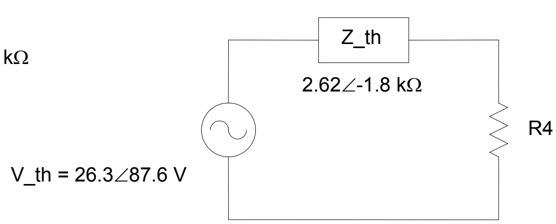


Figure 5 (c)